ARYAN SCHOOL OF ENGINEERING & TECHNOLOGY

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LECTURE NOTE

SUBJECT NAME- FLUID MECHANICS

BRANCH – MECHANICAL ENGINEERING

SEMESTER – 4TH SEM

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1.0 Properties of Fluid:—

1.1 Defination and Units of density, specific weight, specific gravity, Specific Volume.

1.2 Defination and Units of dynamic Viscosity, kinematic Viscosity, Surface tension, Capillary Phenomenon.

Introduction of Fluid Mechanics and Hydraulic Machine:—

Fluid Mechanics Is that branch of Science which deals with the behaviour of the Fluid [liquid or gases] at rest as well as In motion.

It is up off three types:—

i) Static Fluid ii) Dynamic Fluid. iii) kinematic Fluid.

1. Fluid static:—

The branch of Science deals with the static, kinematics and dynamic aspect of Fluid.

The study of Fluid at rest is called static Fluid.

2. kinematic Fluid:—

The study of Fluid In motion, where pressure Force are not Considered Is called kinematic Fluid.

3. Dynamic Fluid:—

The study of Fluid Is In motion, where pressure Force are Considere Is called dynamic Fluid.

Properties of Fluid:—

1. Mass density:— Mass density of a Fluid Is defined as the ratio of the mass of the Fluid to its Volume.

$$\text{Mass density} = \frac{\text{Mass}}{\text{Unit Volume of Fluid}}$$

It is denotted by the symbol ρ [rcho]

Unit $\rightarrow$ kg/m³ In SI

The density of liq may be considere as Constant while that of gases changes with the Variation of pre and temp.

Mathematically —

$$\text{Mass density}(\rho) = \frac{\text{Mass of Fluid}}{\text{Volume of Fluid.}}$$

The value of density of watere Is $1\,gm/cm^3$. orc $1000\,kg/m^3$

2. Specific Weight orc weight density :—

Specific weight orc weight density of a Fluid Is the reatio betn the weight of a Fluid to its Volume.

→ Thus weight perc Unit Volm of a Fluid Is called weight density

→ denotted by the symbol "w".

Thus mathematically $w = \frac{\text{Weight of the Fluid}}{\text{Volm of the Fluid.}}$

$$= \frac{[\text{Mass of the Fluid}] \times [\text{Accelercation due to g}]}{\text{Volume of the Fluid.}}$$

$$= \frac{\text{Mass of the Fluid} \times g}{\text{Volume of the Fluid.}}$$

$$= \rho \cdot g.$$

$$\boxed{w = \rho \cdot g}$$

The Value of specific weight or weight density (w) for water is 9.81×1000 Newton/m^3 in SI Units.

3. Specif Volume:—

Specif Volume of a Fluid is defined as the Volume of a Fluid occupied by a Unit mass or Volume per Unit mass of a Fluid is called specific Volume.

Mathematically —

$$\text{Specific Volume} = \frac{\text{Volume of Fluid}}{\text{Mass of Fluid}}$$

$$= \frac{1}{\dfrac{\text{Mass of Fluid}}{\text{Volm of Fluid}}} = \frac{1}{\rho}$$

The specif Volume is the reciprocal of mass density.

Unit — m^3/kg.

Notes:— It is Commonly applied to gases.

4. Specific gravity:—

Specific gravity is defined as the ratio of the weight density of a Fluid to the weight density of a standard Fluid.

→ For liquid, the standard Fluid is taken water ~~and for~~

→ For gases the standard Fluid is taken as air.

→ Specific gravity is also called relative density.

→ dimensionless quantity.

→ denotted by the symbol "S".

Mathematically:—

$$S\,[\text{for liquid}] = \frac{\text{Weight density of liq}}{\text{weight density of water}}.$$

$$S\,[\text{for gases}] = \frac{\text{Weight density of gas}}{\text{Weight density of air}}.$$

Thus weight density of a liq = $S \times$ weight density of water.

$= S \times 9.81 \times 1000\ N/m^3$.

density of liquid $= S \times$ density of water

$= S \times 1000\ kg/m^3$

If the specific gravity of ~~liq~~ Fluid is known, then the densit[y] of the Fluid will be equal to specific gravity of Fluid multiplied by density of water.

Notes:— Specific gravity of mercury is 13.6

density of mercury is 13.6×1000

$= 13600\ kg/m^3$.

Eg. 1 Calculate the specific weight, density and specific grav[ity] of 1 litre of a liquid which weight 7N.

Soln → Data given as —

Volume 1 litre $= \frac{1}{1000}\ m^3$.

Weight 7 N =

i) Specific weight — $\frac{\text{Weight}}{\text{Volume}} = \frac{7N}{(1/1000)\ m^3}$

$= 7000\ N/m^3$

(ii) Density (ρ) — $\frac{W}{g} = \frac{7000}{9.81}$ kg/m3 $= 713.5$ kg/m3.

(iii) Specific gravity $= \frac{\text{Density of liquid}}{\text{Density of water}} = \frac{713.5}{1000} = 0.713$.

Eg.2 Calculate the density specif weight and weight of 1 litre of petrol of specific gravity 0.7.

Soln → Data given as —

Volume of petrol = 1 litre.

$= 1 \times 1000 \ cm^3$

$= \frac{1000}{10^6} m^3 = 0.001 \ m^3$

Specific gravity $S = 0.7$

i) Density (ρ) $= S \times 1000$ kg/m3 $= 0.7 \times 1000 = 700$ kg/m3.

ii) Specific weight (w) $= \rho \times g = 700 \times 9.81 = 6867 \ N/m^3$.

iii) Weight (W), We know that —

Specific weight (w) $= \frac{\text{Weight}}{\text{Volume}}$.

$w = \frac{W}{0.001}$

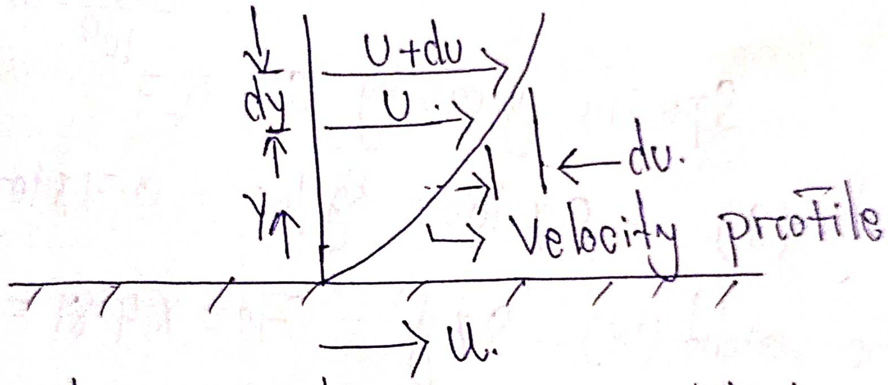
$6867 = \frac{W}{0.001}$

$W = 6867 \times 0.001 = 6.867 \ N$.

Viscosity :-

Viscosity Is defined as the property of a Fluid which offers resistance to the movement of one layer of Fluid over another adjacent layer of the Fluid. When two layers of a Fluid a distance "dy" apart move one over the other at different velocities. Say u and $u + du$.

→ The Viscosity together with relative velocity Causes a Shear stress acting betn the Fluid layers.



[Velocity Variation near a Solid boundary]

→ The top layer Causes a shear stress On the adjacent lower layer while the lower layer Causes a shear. Stress on the adjacent top layer.

→ This shear stress Is proportional to the rate of change of Velocity with respect to y.

→ It is denotted by the symbol τ (Tau).

Mathematically $\tau \propto \frac{du}{dy}$.

$$\tau = \mu \cdot \frac{du}{dy}$$

Where μ = Constant of proportionality and is known as dynamic viscosity or only Viscosity.

From equation we have —

$$\mu = \frac{\tau}{\left(\frac{du}{dy}\right)}$$

Thus viscosity is also defined as the shear stress require to produce unit rate of shear strain.

Units of Viscosity: —

$$\mu = \frac{\text{Shear stress}}{\left(\frac{\text{Change of Velocity}}{\text{Change of distance}}\right)} = \frac{\text{Force/Area}}{\left(\frac{\text{Length}}{\text{time}}\right) \times \frac{1}{\text{Length}}}$$

$$= \frac{\text{Force} \times (\text{Length})^2}{1/\text{time}} = \frac{\text{Force} \times \text{time}}{(\text{Length})^2}$$

i) MKS Unit of Viscosity — $\frac{\text{kgf-sec}}{m^2}$

ii) CGS Unit of Viscosity — $\frac{\text{dyne-sec}}{cm^2}$

iii) SI Unit of Viscosity — $\frac{\text{Newton-Sec}}{m^2} = \frac{N.s}{m^2}$

Notes: —

The unit of Viscosity In CGS Is also called poise.

$$\boxed{1 \text{ poise} = 1 \frac{\text{dyne-sec}}{cm^2}}$$

The numerical Conversion of the Unit of Viscosity from MKS Unit to CGS Unit is given below.

$$\text{One } \frac{kgf\text{-}Sec}{m^2} = \frac{9.81\ N\text{-}Sec}{m^2} \qquad [1\ kgf = 9.81\ N]$$

$1\ N = 1\ kg\ (\text{Mass}) \times 1\ (m/s^2)$ acceleration.

$$= \frac{1000\ gm \times 100\ cm}{Sec^2} = 1000 \times 100\ \frac{gm\text{-}cm}{Sec^2}$$

$$= 1000 \times 100\ \text{dyne.}$$

$$\left[\text{dyne} = \frac{gm\text{-}cm}{Sec^2}\right]$$

$$\frac{1\ kgf\text{-}Sec}{m^2} = 9.81 \times 10,0000\ \frac{dyne\text{-}sec}{cm^2}$$

$$= 9.81 \times 100000\ \frac{dyne\text{-}sec}{100 \times 100 \times cm^2}$$

$$= 98.1\ \frac{dyne.\ sec}{cm^2}$$

$$= 98.1 \text{ poise.}$$

Thus for solving numerical problems, if Viscosity is given in poise, it must be divided by 98.1 to get its equivalent numerical Value in MKS.

$$\text{One } \frac{kgf\text{-}Sec}{m^2} = \frac{9.81\ Ns}{m^2} = 98.1 \text{ poise.}$$

$$\text{One } \frac{Ns}{m^2} = \frac{98.1}{9.81} \text{ poise} = 10 \text{ poise.}$$

$$\boxed{1 \text{ poise} = \frac{1}{10}\ \frac{Ns}{m^2}}$$

Notes:

i) In SI Unit Second is represented by "S" and Not "Sec".

ii) IF Viscosity Is given In poise, it must be divided by 10 to get its equivalant numerical Value In SI Units.

Sometimes a unit of Viscosity as centipoise Is used where

$$1 \text{ centipoise} = \frac{1}{100} \text{ poise.}$$

$$\boxed{1 \text{ c.p} = \frac{1}{100} \text{ P}}$$

The viscosity of water at 20°C Is 0.01 poise or 1.0 Centipoise.

Kinematic Viscosity:

It is defined as the ratio betn the dynamic Viscosity and density of Fluid.

→ It is denoted by the greek symbol "ν" Called "nu".

Mathematically – $\nu = \frac{\text{Viscosity}}{\text{Density}} = \frac{\mu}{\rho}$

Thus Units of kinematic viscosity Is obtained as:

$$\nu = \frac{\text{Units of } \mu}{\text{Units of } \rho} = \frac{\text{Force} \times \text{Time}}{(\text{Length})^2 \times \frac{\text{Mass}}{(\text{Length})^3}}$$

$$= \frac{\text{Force} \times \text{time}}{\text{Mass}/\text{Length}} = \frac{\text{Mass} \times \frac{\text{Length}}{(\text{time})^2} \times \text{time}}{[\text{Mass}/\text{Length}]}$$

$$= (\text{Length})^2/\text{Time}.$$

In MKS and SI, the unit of kinematic Viscosity is $\frac{(\text{Meter})^2}{S}$

CGS Unit → cm^2/Sec.

Notes:—

* In CGS units, kinematic Viscosity is also known as stoke.

Thus One stoke $= cm^2/s = \left(\frac{1}{100}\right)^2 m^2/s$

$= 10^{-4}\ m^2/sec.$

Centistoke means $= \frac{1}{100}$ stroke.

Surface Tension and Capillarity :—

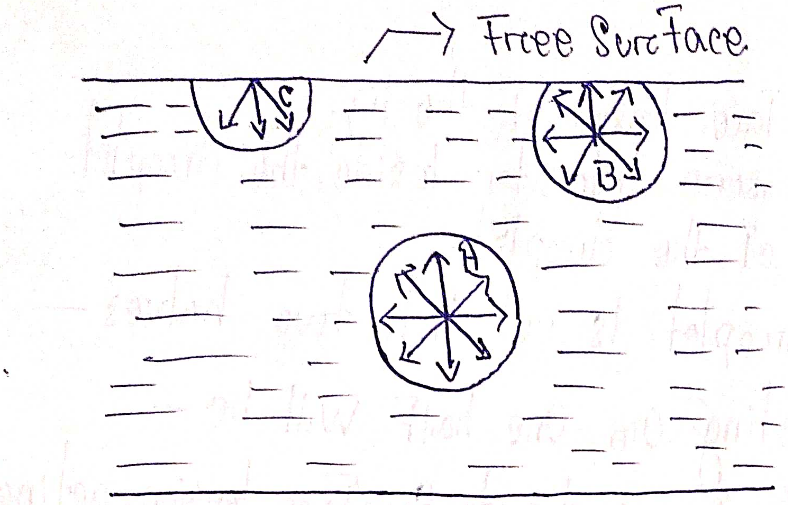
Surface tension is defined as the tensile force acting on the surface of the liquid in contact with a gas or on the surface betn two immiscible liquid such that the contact surface behaves like a membrane under tension.

→ The magnitude of this force per unit length of the free surface will have the same value as the surface energy per unit area.

→ it is denoted by the greek letter "σ"

Units – In MKS → kgf/mtr.

In SI → N/mtr.



→ Consider three molecules A, B, C of a liq In a mass of liq.

→ The molecule A Is attracted In all directions equally by the Surrounding molecules of the liq.

→ Thus the resultant Force acting On the molecule A Is zero.

→ Molecule "B" which Is situated near the Free Surface Is acted upon by upward and downward Forces which are Unbalanced.

→ A net resultant Force on molecule "B" Is acting In the downward direction.

→ The molecule "C" Situated on the Free Surface of liq does experience a resultant downward Force.

→ All the molecules On the Free Surface experienced a downward Force.

→ Thus the Free Surface of the liq acts like a very thin Film Under tension of the Surface of the liq act as through it is an elastic membrane under tension.

Surface tension on liq Droplet:—

Consider a small spherical droplet of a liq of radius "r" On the entire Surface of the droplet. the tensile Force due to Surface tension will be acting.

Let σ = Surface tension of the liq.
P = pressure Intensity Inside the droplet.
d = dia of the droplet.

Let the droplet is cut. Into two halves —

The Force acting on One half will be —

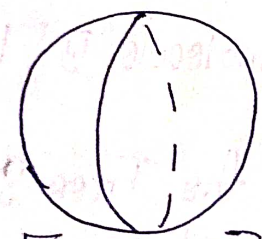
i) Tensile force due to Surface tension. acting around the Circumference of the Cut portion.

$= \sigma \times$ Circumference

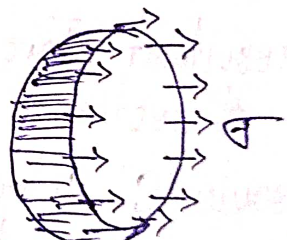
$= \sigma \times \pi d$.

(ii) pressure Force on the area —

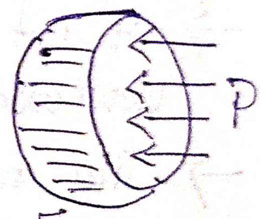
$$\frac{\pi}{4} d^2 = P \times \frac{\pi}{4} d^2$$



[Droplet]



[Surface tension]



[Pressure Force]

These two Forces will be equal & opposite Under equlibrium Condition.

$$P \times \frac{\pi}{4} d^2 = \sigma \times \pi d.$$

$$P = \frac{\sigma \times \pi d}{\frac{\pi}{4} d^2} = \frac{4\sigma}{d}.$$

Surface tension in a hollow bubble:—

A hollow bubble like a soap bubble in air has two surface in contact with air. One inside and other outside.

Thus two surfaces are subjected to surface tension.

$$P \times \frac{\pi}{4} d^2 = 2 \times (\sigma \times \pi d)$$

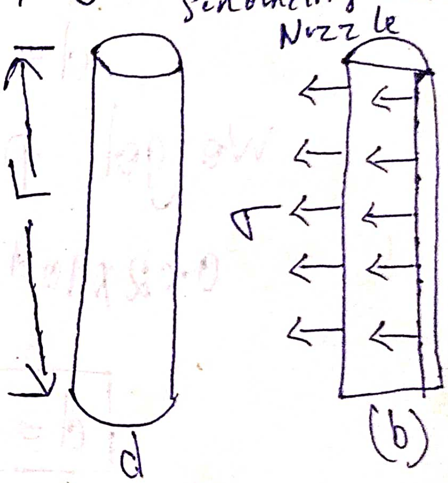
$$P = \frac{2 \times \sigma \times \pi d}{\frac{\pi}{4} d^2} = \frac{8\sigma}{d}.$$

Surface tension on a liq jet:— [stream of Fluid that is projected into a surrounding Medium]

Consider a liq jet of diameter "d" and length "L".

Let p = pre intensity inside the liq jet above the outside pressure.

σ = Surface tension of the liq.



Consider the equlibrium of the semi jet, we have —

Force due to pre = p x area of semi jet
= $p \times L \times d$.

Force due to surface tension = $\sigma \times 2L$

Equating the force,

$$p \times L \times d = \sigma \times 2L$$

$$p = \frac{\sigma \times 2L}{L \times d}.$$

Eg. 1 The surface tension of water in contact with air at 20°C is 0.0725 N/m. The pr inside a droplet of water is to be 0.02 N/cm² greater than the outside pr. Calculate the diameter of the droplet of water.

Soln → Data given as —

Surface tension (σ) = 0.0725 N/m.

Pressure intensity P in excess of outside pressure is —

$$P = 0.02 \ N/cm^2 = 0.02 \times 10^4 \ \frac{N}{m^2}$$

d = dia of the droplet.

We get $P = \frac{4\sigma}{d}$

$$0.02 \times 10^4 = \frac{4 \times 0.0725}{d}$$

$$\boxed{d = 1.45 \ mm.}$$

Eg. 2 Find the surface tension in a soap bubble of 40 mm diameter when the inside pressure is 2.5 N/m² above atmospheric pressure.

Soln → Data given as —

dia of bubble $d = 40$ mm $= 40 \times 10^{-3}$ m

pressure in excess of outside $P = 2.5 \ N/m^2$

For a soap bubble, using eqn. we get —

$$P = \frac{8\sigma}{d}$$

$$2.5 = \frac{8 \times \sigma}{40 \times 10^{-3}} \Rightarrow \sigma = \frac{2.5 \times 40 \times 10^{-3}}{8} \text{ N/m.}$$

$$= 0.0125 \text{ N/m.}$$

Eg.3 The prc outside the droplet of water of dia 0.04 mm is 10.32 N/cm². Calculate the prc within the droplet if Surface tension is given as 0.0725 N/m of water.

Soln → Data given as —

dia of the droplet — $d = 0.04 \text{ mm} = 0.04 \times 10^{-3}$ m.

pressure outside the droplet $= 10.32 \text{ N/cm}^2$.

$= 10.32 \times 10^4 \text{ N/m}^2$.

Surface tension. $\sigma = 0.0725$ N/m.

The prc inside the droplet, in excess of outside prc is given by eqn.

$$P = \frac{4\sigma}{d} = \frac{4 \times 0.0725}{0.04 \times 10^{-3}} = 7250 \text{ N/m}^2.$$

$$= \frac{7250}{10^4 \text{ cm}^2} = 0.725 \text{ N/cm}^2.$$

Pressure inside the droplet —

$= P +$ prc. outside the droplet.

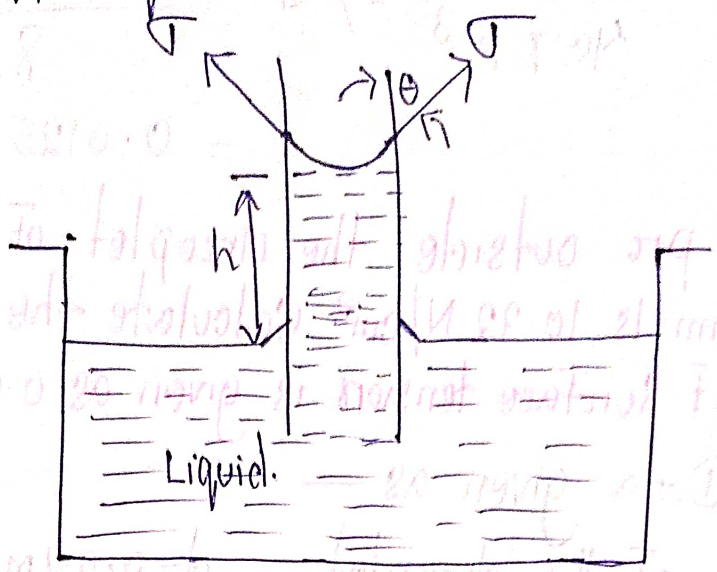
$= 0.725 + 10.32$

$= 11.045 \text{ N/cm}^2$.

Capillarity :—

→ Cappilarcity is defined as a phenomenon of rise or fall of a liq surface in a small tube relative to the adjacent general level of liq when the tube is held

Verctically In liq.



The rcise of liq Surcface Is known as cappilarcy rcise while the fall of the liq Surcface Is known as cappilarcy depression.

→ Unit - cm orc mm of liq.

→ Its value depends upon the i) specific weight of the liq
ii) diametere of the tube.
iii) Surcface tension of the liq.

Assignment

i) what is mass and weight density.
ii) what do you mean by FMHM and its classification.
iii) what is specific Volume.
iv) what is specific gravity.
v) what Is Viscosity.
vi) what Is kinematic Viscosity.
vii) what Is Surcface tension.
viii) what Is capillarcity.

Chapter-II Fluid pressure & its measurement

Syllabus: —

2.1 Defination and Units of Fluid prc, prc Intensity and prc head.

Fluid prc at a point: —

Consider a small area dA In large mass of Fluid.

→ IF the Fluid Is stationary then the Force exerted by the Surrounding Fluid On the area dA will always be perpendicular to the Surface dA.

→ Let "dF" Is the Force acting On the area dA In the normal direction.

→ Then the ratio of dF/dA Is known as the Intensity of prc

→ it is represented by "p".

$$\boxed{p = dF/dA}$$

IF the Force Is Uniformly distributed over the area. (A) then prc at any point Is given by —

$$p = \frac{F}{A} = \frac{\text{Force}}{\text{Area}}$$

Force or prc Force = $F = P \times A$

Units of prc are -

1) kgF/m2 and kgF/cm2 — M.k.S.
2) N/m2 or Newton/m2 — SI

N/m2 Is known as pascal represented by "Pa"

kpa = kilo pascal = 1000 N/m2.

1 bar = 100 kpa = 105 N/m2.

Q.1 A hydraulic press has a ram of 30cm diameter and a plunger of 4.5 cm diameter. Find the weight lifted by the hydraulic press when the Force applied at the plunger Is 500 N.

Solution ⟶ Data given as —

dia of ram = D = 30 cm = 0.3m.

" " plunger d = 4.5 cm = 0.045 m.

Force on plunger F = 500 N.

Weight = W

Area of ram = $A = \frac{\pi}{4} D^2 = \frac{\pi}{4} (0.3)^2 = 0.0706 \, m^2$

Area of plunger $a = \frac{\pi}{4} d^2 = \frac{\pi}{4} (0.045)^2 = .00159 \, m^2$

Prc Intensity due to plunger = $\frac{\text{Force of plunger}}{\text{Area of plunger}}$

$= \frac{500}{.00159} \, N/m^2$.

due to pascal's law the Intensity of prc will be equally transmitted In all direction.

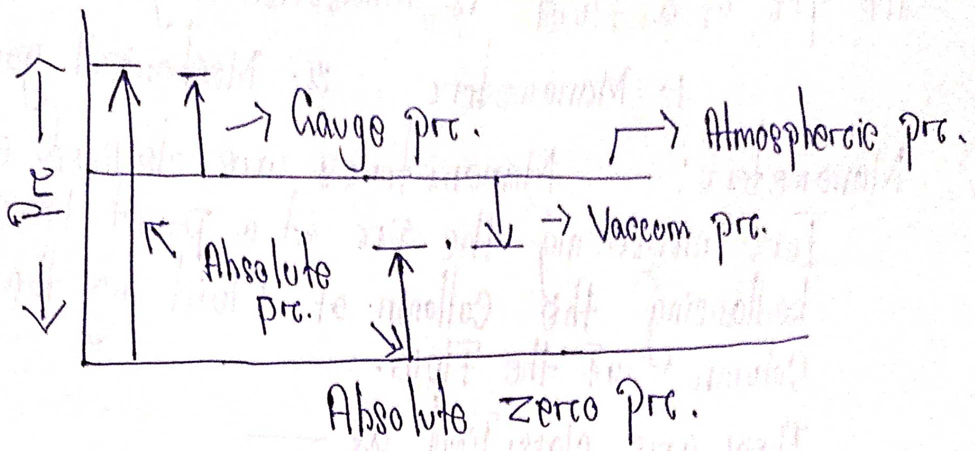
Pre Intensity of ram $= \dfrac{500}{.00159} = 314465.4\ N/m^2.$

Pre Intensity at ram $= \dfrac{Wt}{\text{Area of ram}} = \dfrac{W}{A} = \dfrac{W}{.07068}\ \dfrac{N}{m^2}$

$$\frac{W}{0.07068} = 314465.4.$$

$$\boxed{W = 22.222\ kN.}$$

Absolute, Gauge, atmospheric and Vaccum prc:—



i) Atmospheric Prc:—

The atmospheric air exerts a normal prc upon all Surfaces with which it is in contact and known as atm prc.

ii) Absolute prc:—

It is defined as the prc which is measured with reference to absolute vaccum prc. or absolute zero prc.

iii) Gauge prc:—

It is defined as the prc which is measured with the help of a prc measuring instument in which the atm prc is ~~less~~ taken as datum. the atm prc in the scale is marked as zero.

iv) Vaccum prc:—

It is defined as the prc below the atm prc.

Mathematically:—

abs prc = atm prc + gauge prc.

$$P_{abs} = P_{atm} + P_{gauge}.$$

Vaccum prc = atm prc − abs prc.

$$P_{vaccum} = P_{atm} - P_{abs}.$$

Pressure measuring Instument :—

The prc of a Fluid Is measured by Following devices –

1. Manometerc 2. Mechanical gauge.

i) Manometerc :— Manometercs are defined as the device fore measuring the prc at a point In a Fluid by ballancing the collomn of Fluid by the Same another Column of the Fluid.

They are classified as —

i) Simple Manometerc. ii) Differential Manomet

2) Mechanical gauge :— Mechanical gauges are defined as the device used fore measuring the prc by balancing the Fluid Column by the spring or dead weight, Commonly Used mechanical prc gauges are —

i) Diphagream Prc gauge.
ii) Burdon tube prc gauge.
iii) Dead weight prc gauge.
iv) Bellow prc gauge.

Simple Manometerc :— A Simple manometerc of a glass tube having One of its ends Connected to a point where prc Is to be measured and other end remains open to atm.

Common type of Simple Manometercs are —

i) piezometerc.
ii) U-tube Manometerc.
iii) Single column Manometerc.

Pie-
→
→
→
→
U-
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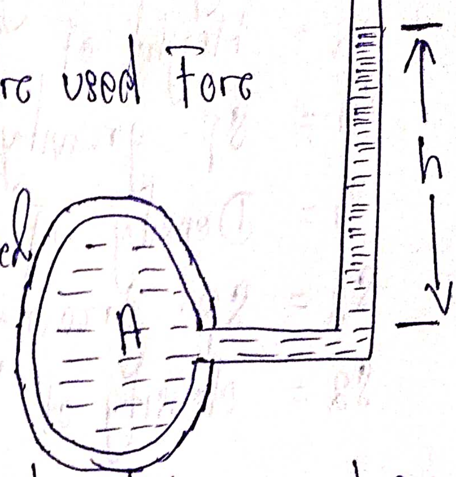
→

Piezometer :—



→ It is the simple Form of Manometer used For measuring gauge prc.

→ One end of the Manometer Is Conected to the point where prc Is to be measured and other end is open to the atm.

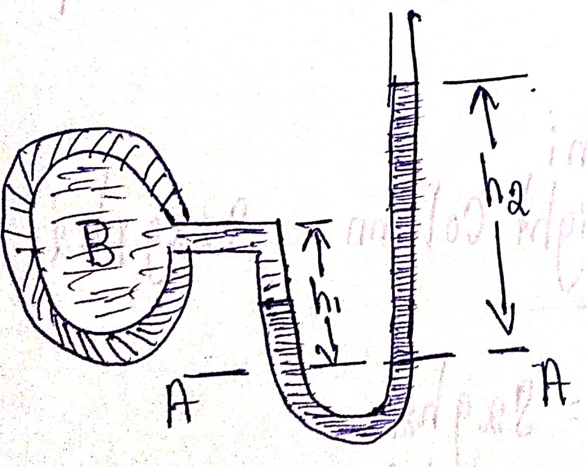
→ The rise of liq gives the prc head at the point A.

→ Then prc at A $\boxed{P_A = \rho \cdot g \cdot h}$

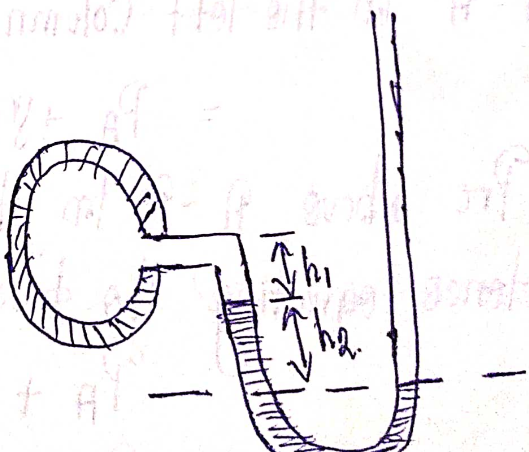
U-tube Manometer :—

It Consist of glass tube bent In U-shape. One end of which Is Connected to a point at which prc Is to be measured and other end remains open to the atm.

→ The tube generally Contains mercury.



@ For gauge prc.

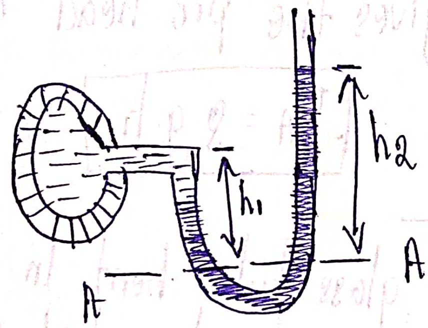


(b) For Vaccum Prc.

@ For gauge prc :—

Let be the point which Is to be measured, whose value Is P. The datum line Is A-A.

Let h_1 = Height of light liq above the datum line.
h_2 = Height of heavy liq above the datum line.
S_1 = Sp. gravity of liq light.
ρ_1 = Density of light liq = $1000 \times S_1$
S_2 = Sp. gravity of heavy water.
ρ_2 = density of heavy weight = $1000 \times S_2$.



(a) For gauge prc.

Prc Is Same In a horizontal Surface. Hence prc above the horizontal datum Surface line A-A In the left Column and In the right Column of U-tube manometer should be Same prc above A-A In the left Column.

$$= P_A + \rho_1 \times g \times h_1$$

Prc above A-A In the right Column = $\rho_2 \times g \times h_2$.

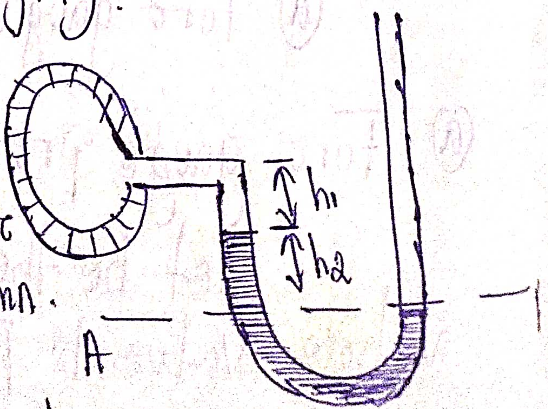
Hence equating the two prc —

$$P_A + \rho_1 g h_1 = \rho_2 g h_2.$$

$$P_A = (\rho_2 g h_2 - \rho_1 g h_1).$$

(b) For Vaccum Prc :—

For measuring Vaccum Prc the level of the heavy liq In the manometer Then prc above A-A In the left Column.



$$\rho_2 g h_2 + \rho_1 g h_1 + P_A$$

Prc head In the right Column above A-A ...

Prc above A-A In the left Column —

$$\rho_2 g h_2 + \rho_1 g h_1 + p$$

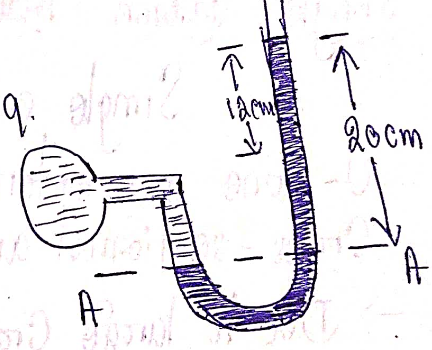
Prc head In the right column above A-A = 0

$$\rho_2 g h_2 + \rho_1 g h_1 + p = 0$$

$$p = -[\rho_2 g h_2 + \rho_1 g h_1]$$

Q: The right limb of a simple U-tube manometer containing mercury is open to the atmosphere while the left limb is connected to a pipe in which a fluid of sp grc is 0.9. The centre of the pipe is 12 c.m below the level of mercury in the right limb. Find the prc of fluid in the pipe if difference of mercury level in the two limbs is 20 c.m.

Solⁿ → Data given as —



SP gravity of Fluid $S_1 = 0.9$.

Density of Fluid $= \rho_1 = S_1 \times 1000$

$= 0.9 \times 1000$

$= 900 \ kg/m^3$.

SP grc of mercury $= 13.6$

Density of mercury $= \rho_2 = 13.6 \times 1000 = 13600 \ kg/m^3$.

Difference of mercury level $h_2 = 20 cm = 0.2m$.

Height of Fluid From A-A $h_1 = 20 - 12 = 8cm = 0.08m$.

Let p = Prc of Fluid In pipe.

Equating the above prc. we get —

$$p + \rho_1 g h_1 = \rho_2 g \cdot h_2.$$

$$p + 900 \times 9.81 \times 0.08 = 13.6 \times 1000 \times 9.81 \times .2$$

$$P = (13.6 \times 1000 \times 9.81 \times 0.2) - (900 \times 9.81 \times 0.08)$$
$$= 25977 \text{ N/m}^2.$$
$$= 2.597 \text{ N/cm}^2. \quad \underline{\text{Ans}}$$

Q. 2 **Assignment:**

A Small U-tube manometer containing mercury is connected to a pipe in which fluid of sp. gr. 0.8. and having vaccum prc is flowing. The other end of the manometer is open to atm. Find the vaccum prc in pipe, If the difference of mercury level in the two limb is 40cm and the height of fluid in the left from centre of pipe is 15 c.m below.

Single column Manometer:—

Single column Manometer is a modified form of U-tube manometer. In which a reservoir having a large cross-sectional area.

→ Due to large cross sectional area of the reservoir, for any variation of prc the change in the liq level in the reservoir will be very small which may be neglected and hence the prc is given by the height of liq in the other limb.

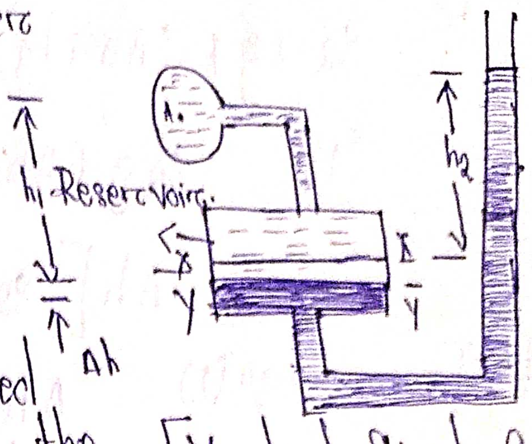
→ The other limb may be vertical or inclined.

→ Thus there are two types of Single column Manometer-

i) Vertical Single column Manometer.
ii) Inclined Single column Manometer.

1) Vertical Single column Manometer :-

Vertical Single column Manometer



[Vertical Single Column Manometer]

Let X-X be the datum line In the reservoir and in the right limb of the manometer. when it is not Conected to the pipe.

→ when the manometer Is Conected to the pipe, due to high prc at A. the heavy liq In the reservoir will be pushed downward and will rise In right limb.

Δh = Fall of heavy liq In reservoir.

h_2 = Rise of heavy liq In right limb.

h_1 = Height of Centre of pipe above X-X.

P_A = Prc at A, which Is to be measured.

A = Cross-sectional area of the reservoir.

a = Cross Sectional area of the light limb.

S_1 = Sp gravity of liq In pipe.

S_2 = Sp gravity of heavy liq In reservoir & right limb.

ρ_1 = Density of liq In pipe.

ρ_2 = density of liq In reservoir.

Fall of heavy liq In reservoir will Cause a rise of heavy liq level In the right limb.

$$A \times \Delta h = a \times h_2.$$

$$\boxed{\Delta h = \frac{a \times h_2}{A}} \quad \text{—(1)}$$

Now Consider the datum line Y-Y as shown In Fig.

Then prc In the right limb above Y-Y = $\rho_2 \times g \times (\Delta h + h_2)$

Prc above the left limb above Y-Y = $\rho_1 \times g \times (\Delta h + h_1)$ + ...

Equating these pre we have :—

$$S_2 \times g \times (\Delta h + h_2) = S_1 \times g \times (\Delta h + h_1) + P_A$$

$$P_A = S_2 g (\Delta h + h_2) - S_1 g (\Delta h + h_1)$$

$$= \Delta h [S_2 g - S_1 g] + h_2 S_2 g + h_1 S_1 g.$$

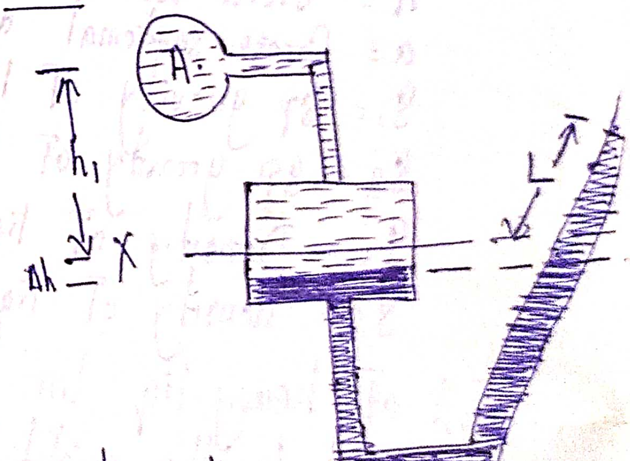
but From eqn (1) $\Delta h = \frac{a \times h_2}{A}$

$$P_A = \frac{a \times h_2}{A} [S_2 g - S_1 g] + h_2 S_2 g - h_1 S_1 g.$$

As the area A Is very large as compaired to a, hence ratio a/A becomes very small and can be neglected.

$$\boxed{P_A = h_2 S_2 g - h_1 S_1 g}$$

Inclined Single Column Manometere :—



Inclined Single Column Manometere Is more Sensitive.

→ Due to Inclination the distance moved by the heavy liq In the right limb will be more.

Let L = Length of heavy liq moved In the right limb.
θ = Inclination of right limb with horizontal.
h_2 = Vertical rise of heavy liq In right limb from x.
$= L \times \sin\theta$.

From the eqn the pre at A —

$$P_A = h_2 S_2 g - h_1 S_1 g.$$

Substituting the value of h_2 we get.

Q. 1 A Single column Manometer Is Connected to a pipe Containing a liq of sp. gr 0.9. Find the pr In the pipe IF the area of the reservoir Is 100 times the area of the tube For the manometer reading. The sp gr of mercury Is 13.6.

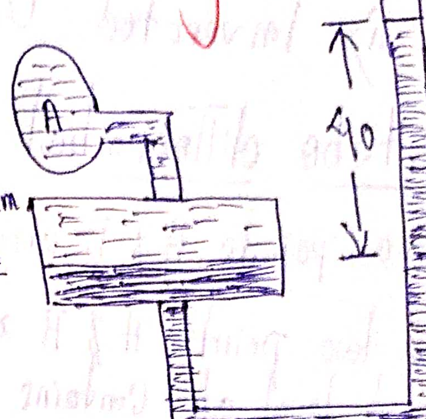
Solution → Data given as —

$S_1 = 0.9$

$\rho_1 = 900\ kg/m^3$

$S_2 = 13.6$

$\rho_2 = 13.6 \times 1000$ ← Density



$$\frac{\text{Area of reservoir}}{\text{Area of the right limb}} = \frac{A}{a} = 100.$$

Height of the liq $h_1 = 20\ c.m. = 0.2 m.$

Rise of mercury In the right limb $= h_2 = 40\ c.m = 0.4\ m.$

P_A = Pr In Pipe.

Using eqn we get —

$$P_a = \frac{a}{A} h_2[\rho_2 g - \rho_1 g] + h_2 \rho_2 g - h_1 \rho_1 g$$

$$= \frac{1}{100} \times 0.4[13600 \times 9.81 - 900 \times 9.81] + 0.4 \times 13600 \times 9.81 - 0.2 \times 900 \times 9.81$$

$= 52134\ N/m^2$

$= 5.21\ N/cm^2.$

Differential Manometer: —

Differential Manometers are the device used for measuring the difference of pr betn two points In a pipe or In two different pipes.

→ A differential manometer Consist of a U-tube Containing a heavy liq whose two ends are Conected to the Points.

whose difference of prc is to be measured.

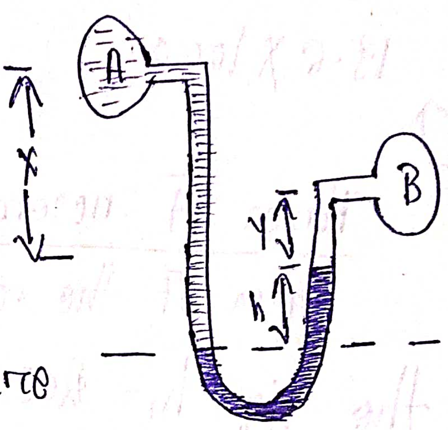
Most Commonly types of differential manometers:—

i) U-tube differential manometer.

ii) Inverted U-tube differential Manometer.

i) U-tube differential Manometer:—

Two points A & B are at different level—



Let the two points A & B are at different level also contains liq of different sp. grc.

→ These points are conected to the U-tube differential manometer.

→ Let the prc at point A & B are P_a & P_b.

Let h = difference of mercury level in the U-tube.

y = distance of the centre of B. from the mercury level in the right limb.

S_1 = density of liq at A.

S_2 = " " " at B.

S_g = " " heavy liq are mercury.

Taking datum line at X-X.

Prc above X-X in the limb = $S_1 g(h+x) + P_A$

when prc P_A = Prc at A.

Prc above X-X in the right limb = $S_g \times g \times h + S_2 \times g \times y + P_B$

where prc P_B = Prc at B.

Equating the two prc, we have—

$S_1 g(h+x) + P_A = S_g \times g \times h + S_2 \times g \times y + P_B$.

$P_A - P_B =$

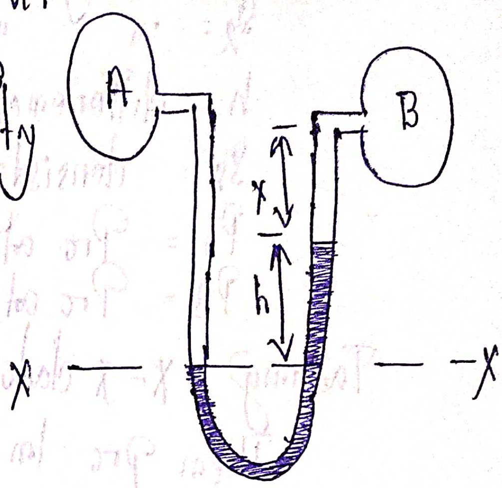
$= h \times g\, [S_g - S_1] + S_2 g y - S_1 g x.$

difference of prc at A & B –

$h \times g\, [S_g - S_1] + S_2 g y - S_1 g x.$

Two points A & B are at same level —

In the given Fig A & B are the same level and contains the same liq of density S_1.



Prc above X-X In right limb —

$S_g \times g \times h + S_1 \times g \times x + P_B$

Prc above X-X In left limb —

$P_A \times g \times (h+x) + P_A.$

Equating the two prc :—

$S_g \times g \times h + S_1 \times g \times x + P_B = P_1 \times g \times (h+x) + P_A$

$P_A - P_B = S_g \times g \times h + P_1 g x - P_1 g \times (h+x).$

$= g \times h\, (P_B - P_A).$

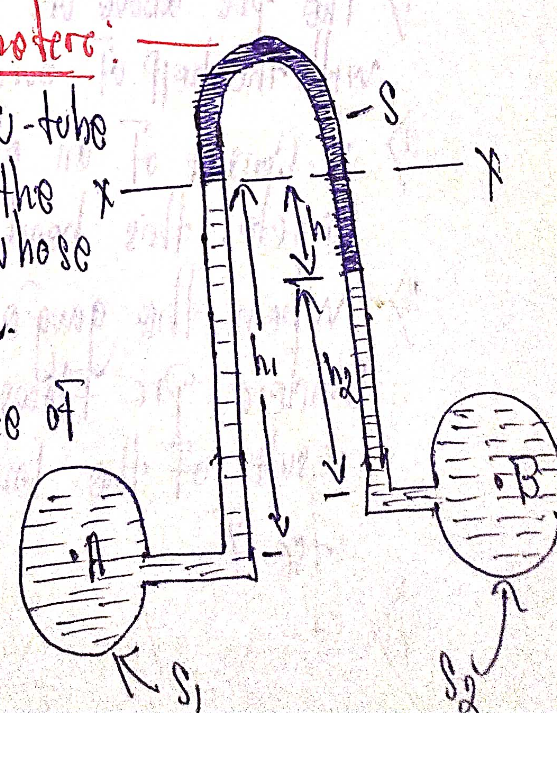
Inverted U-tube differential Manometer :—

It consist of an Inverted U-tube containing a light liq. The two ends of the U-tube are connected to the points whose difference of prc Is to be measured.

→ It is used for measuring difference of low prc.

→ Inverted U-tube manometer connected to the point A & B.

P_B



→ Inverted U-tube manometer connected to the points A & B
Let the pre at A is more than the pre at B.

Let h_1 = Height of liq in the left limb below datum line X-X.
h_2 = Height of liq in the right limb.
S_1 = density of liq at A.
S_2 = " " " " B.
h = difference of light liq.
S_s = density of light liq.
P_A = Pre at A.
P_B = Pre at B.

Taking X-X datum line:—
Then pre in the left limb below X-X $= P_A - S_1 g h_1$
" " " right " " X-X $= P_B - S_2 g h_2 - S_s g h$

Equating the two pre —
$$P_A - S_1 g h_1 = P_B - S_2 g h_2 - S_s g h.$$
$$P_A - P_B = S_1 g h_1 - S_2 g h_2 - S_s g h.$$

Burdon tube pre gauge:—

1) The pre above or below the atm pre may be easily measured with the help of burdon tube pre gauge.
2) It consist of an eliptical tube ABC bent into an arc of a circle. this bent up tube is called burdon tube pre
3) When the gauge tube is connected to the C, the fluid under pre flows into the tube of the burdon tube as a result of the increased pre ~~tube~~ tends to straighten itself.
4) Since the tube is increased in a circular
Cover therefore it tends ...

$$\frac{P_A - P_B}{S_0 g} = x\left[\frac{Sg}{S_0} - 1\right]$$

$$h = x\left[\frac{S_h}{S_0} - 1\right]$$

Case - II. If the differential manometer contains a liq lighter than the liq flowing through the pipe.

where S_1 = sp gravity of lighter liq in U-tube manometer.
S_0 = " " " fluid flowing through in U-tube ".
x = difference of lighter liq columns in U-tube.

The value of h is given by — $h = x\left[1 - \frac{S_1}{S_0}\right]$

Case - III → Inclined Venturimeter with differential U-tube manometer.

Let the differential manometer contains heavier liq.
Then h is given as —

$$h = \left[\frac{P_1}{Sg} + z_1\right] - \left[\frac{P_2}{Sg} + z_2\right] = x\cdot\left[\frac{S_h}{S_0} - 1\right]$$

Case - IV - Similarly for inclined Venturimeter in which differential manometer contains a liq which is lighter than the liq flowing through the pipe. Then -

$$h = \left[\frac{P_1}{Sg} + z_1\right] - \left[\frac{P_2}{Sg} + z_2\right]$$

$$h = x\left[1 - S_1/S_0\right]$$

Limitations:—

1) Bernoullis eqn has been derived under the assumption that no external force except the gravity force is acting on the liq. But in actual practice some external force always acting on the liq when effect the flow of liq.

2) If the liq is flowing in a curved path the energy due to centrifugal force should also be taken into account.

pitot - tube: — It is a device used for measuring the velocity of fl at any point in a pipe or a channel.

→ It is based on the principle that if the velocity flow at a point becomes zero, the prc there is increased due to conversion of the kinetic energy into prc energy.

The pitot tube consist of a glass tube bent an right angle. Consider two points 1 & 2 at the same level such as stage 2 is the inlet of pitot-tube and 1 is the far away from tube.

Let P_1 = prc at point 1.

V_1 = Vel of fluid at point 1.

P_2 = Prc at point 2.

V_2 = Vel of fluid at point 2.

H = Depth of tube in the liq.

h = Rise of the liq in the tube above free surfa

Applying Bernoullis theorem —

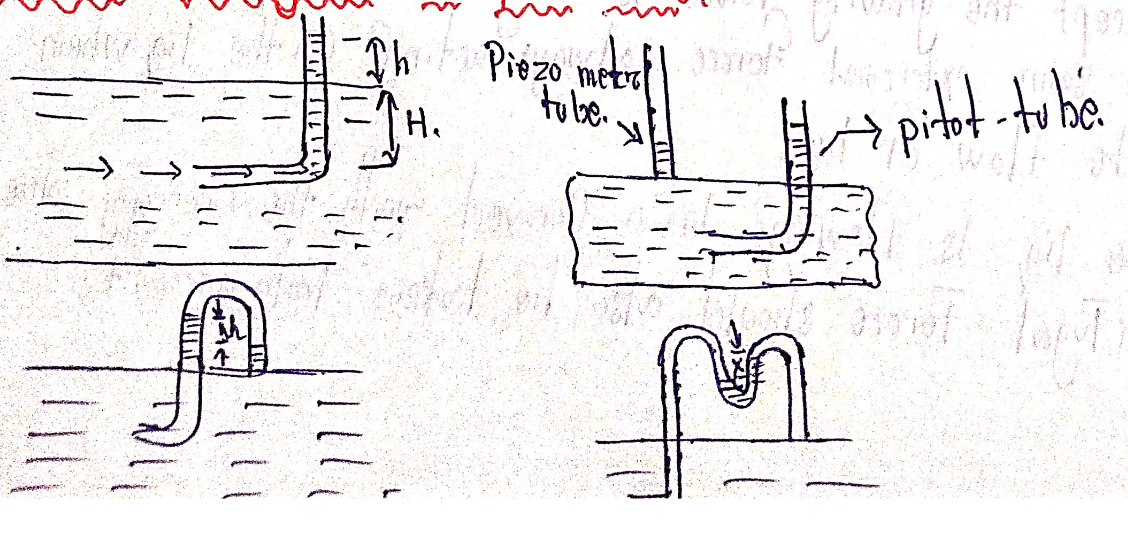
$$P_1/\rho g + V_1^2/2g + Z_1 = P_2/\rho g + \frac{V_2^2}{2g} + Z_2 \qquad \boxed{\frac{P_1}{\rho g} = H, \ \frac{P_2}{\rho g} = h+}$$

$$H_1 + \frac{V_1^2}{2g} + \cancel{Z_1} = h + H.$$

$$V_1 = \sqrt{2gh}.$$

Actual Velocity $V_{act} = C_v\sqrt{2gh}$. where C_v = Co-efficient of pitot.

Different arrangement of pitot-tube: —



Eg.1 Water is flowing through a pipe of 5cm dia under prc of 29.43 N/cm² and with mean velocity 2m/s. Find the total head or total prc energy per unit weight of the water at a cross-section which is 5m above the datum line.

Sol^n → Data given as:—

dia of pipe (d) = 5cm = 0.5m.

Pressure $P = 29.43\ N/cm^2 = 29.43 \times 10^4\ N/cm^2$

Velocity (V) = 2 m/s.

Datum head Z = 5m.

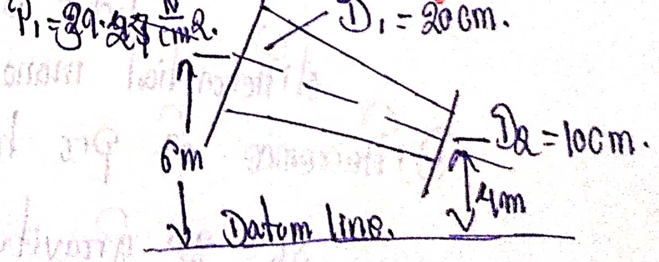
Total head = Prc head + kinetic head + datum head.

$$= \frac{P}{\rho g} + \frac{V_1^2}{2g} + Z_1$$

$$= \frac{29.43 \times 10^4}{1000 \times 9.81} + \frac{2^2}{2 \times 9.81} + 5$$

$$= 35.204\ m.$$

Eg.2 The water is flowing through a pipe having diameter 20 cm & 10cm at sec I & II respectively. The rate of flow through pipe is 35 lit/s. The section I is 6m above datum and sec 2 is 4m above datum. If the prc at sec I is 39.24 N/cm². Find the intensity of prc. at section 2.



Sol^n → At section 1 →

$D_1 = 20cm = 0.2m.$

$A_1 = \frac{\pi}{4}(0.2)^2 = 0.0314 m^2.$

$P_1 = 39.24\ N/cm^2.$

$Z_1 = 6\ m.$

At section - 2. $D_2 = 0.10m$,

$A_2 = \frac{\pi}{4}(0.1)^2 = 0.00785 m^2$

$Z_2 = 4m.$

$P_2 = ?$

Rate of Flow $Q = 35$ lit/s. $= \frac{35}{...} = 0.035\ m^3/s$

Now, $Q = A_1V_1 = A_2V_2$.

$$V_1 = Q/A_1 = 0.35/0.0314 = 1.114 \text{ m/s.}$$

$$V_2 = Q/A_2 = 0.035/0.00785 = 4.456 \text{ m/s.}$$

Applying Bernoullis eqn at Section 1&2, we get —

$$P_1/\rho g + V_1^2/2g + Z_1 = P_2/\rho g + V_2^2/2g + Z_2.$$

$$\frac{39.24 \times 10^4}{1000 \times 9.81} + \frac{(1.114)^2}{2 \times 9.81} + 6 = \frac{P_2}{1000 \times 9.81} + \frac{(4.456)^2}{2 \times 9.81} + 4.$$

$$\boxed{P_2 = 40.27 \text{ N/cm}^2}$$

Eg.3. A horizontal Venturimeter with Inlet and throat diameter 30cm and 15cm respectively is used to measure the flow of water. The reading of differential manometer connected to the Inlet & throat is 20cm of mercury. Determine the rate of flow. Take $C_d = 0.98$.

Soln → Data given as: —

dia of Inlet - $d_1 = 30$cm. $= 0.3$m.

$$a_1 = \frac{\pi}{4}(30)^2 = 706.85 \text{ cm}^2.$$

dia of throat — $d_2 = 15$cm

$$a_2 = \frac{\pi}{4}(15)^2 = 176.7 \text{ cm}^2.$$

$C_d = 0.98$.

differential manometer $= x = 20$ cm of mercury.

Difference of pr head $h = x\left[\frac{S_h}{S_o} - 1\right]$

S_h = sp gravity of mercury = 13.6

S_o = sp " " water = 1

$$h = 20\left[\frac{13.6}{1} - 1\right] = 20 \times 12.6 = 252 \text{ cm of water.}$$

discharge through Venturimeter is given by eqn —

$$Q = C_d \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

$$= 0.98 \frac{706.85 \times 176.7}{\quad} \times \sqrt{2 \times 9.81 \times 252} = 125$$

Orifice: — Orifice is a small opening of any cross-section [such as triangular, rectangular etc] on the side or at the bottom of a tank. through which a fluid is flowing.

→ it is used for measuring the rate of flow of fluid.

Applying Bernoullis theorem at 1 & 2 —

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 , \quad H + 0 = 0 + \frac{V_2^2}{2g} . \quad V_2 = \sqrt{2gh}.$$

Orifice Co-efficient: — The coefficients of orifice are —

1) Co. of Velocity C_v. 2) Co. of Contraction C_c 3) Co of discharge C_d.

i) Co. of Velocity C_v — it is defined as the ratio betn the actual vel of a jet of liq at Vena Contracta and the theoritical vel of jet.

→ denotted by C_v.

$$C_v = \frac{\text{Actual vel of jet at Vena-Contracta}}{\text{Theoritical Velocity.}} = \frac{V}{\sqrt{2gh}}.$$

where V = actual vel.

C_v = Co. of vel.

$\sqrt{2gh}$ = theoritical vel.

C_v range 0.95 to 0.99. For diff orifices depends on shape & size.

ii) Co. of Contraction (C_c) — it is defined as the ratio of the area of the jet at Vena-Contracta to the area of the orifice.

→ denotted by C_c. $\boxed{C_c = \frac{a_c}{a}}$

a = area of the orifice.

a_c = area of the Vena-Contracta. $C_c = \frac{\text{area of the jet at Vena Contracta}}{\text{area of the Orifice.}}$

The value of C_c varies from 0.61 to 0.69 depending on shape & size of orifice.

Co-of discharge: — it is the ratio of actual discharge from an Orifice to the theoritical discharge from the orifice.

→ denotted by C_d.

→ if Q is actual discharge & Q_{th} is theoritical discharge then.

$\boxed{C_d = \frac{Q_{act}}{Q_{th}}}$ $\boxed{C_d = \frac{\text{act vel} \times \text{act area}}{\text{Th. vel} \times \text{Th. area}}}$ $\boxed{C_d = C_v \cdot C_c}$

Range. 0.61 to 0.65 but for general purpose 0.62.

Classification:—

i) Acc to the Size :—

i) Small orifice [if the head of liq above the centre of orifice is more than 5 times the depth of orifice]

ii) Large Orifice [if head is less than 5 times the depth of orifice]

ii) According to shape:—

1. Circular. 2. Triangular 3. Rectangular 4. Square.

iii) According to the shape of upstream edge:—

i. sharp edged orifice ii. Bell mounted orifice.

iv) According to nature of discharge:—

1. Free discharge Orifice.

2. Drowned or submerged Orifices—

i) partially submerged orifice ii) Fully submerged orifice.

Orifice meter or Orifice plate:—

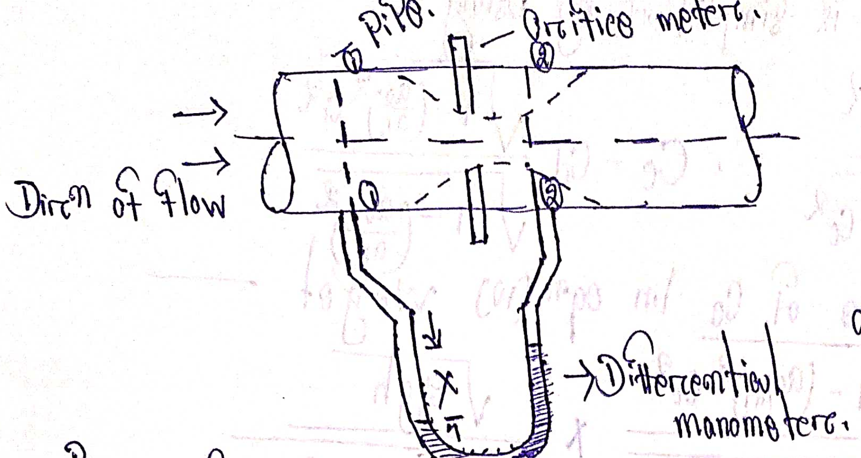
i) It is a device used for measuring the rate of flow of fluid thro a pipe.

ii) It is cheaper as compared to Venturimeter.

iii) It also works as same principle with Venturimeter.

iv) It consist of a flat circular plate which has a sharp edge h called orifice, which is concentric with the pipe.

v) The orifice dia is kept generally 0.5 times the dia of the pipe, through it may vary 0.4 to 0.8 times the pipe dia.



Let P_1 = Pr at Section (1)
V_1 = Vel " "
a_1 = Area of pipe at Section (1).
a_2, V_2, P_2 are corresponding values at Sec (11).

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2.$$

$$\left[\frac{P_1}{\rho g} + Z_1\right] - \left[\frac{P_2}{\rho g} + Z_2\right] = \frac{V_2^2}{2g} - \frac{V_1^2}{2g}.$$

but $\left[P_1/\rho g + Z_1\right] - \left[P_2/\rho g + Z_2\right] = h.$ = differential head.

$$h = \frac{V_2^2}{2g} - \frac{V_1^2}{2g} \quad \text{or} \quad 2gh = V_2^2 - V_1^2. \quad V_2 = \sqrt{2gh + V_1^2}. \text{ — (I)}$$

Now Section (2) is at the Vena Contracta and a_2 represents the area at the Vena Contracta.

If a_0 is the area of Orifice, then we have. $C_c = \frac{a_2}{a_0}$.

Where C_c = Co. of Contraction. $a_2 = a_0 \times C_c$ — (II)

By Continuity eqn — $a_1 V_1 = a_2 V_2$ or $V_1 = \frac{a_2}{a_1} V_2 = \frac{a_0 C_c}{a_1} V_2$ — (III)

Substituting the values of V_1 in eqn (I) we get —

$$V_2 = \sqrt{2gh + \frac{a_0^2 C_c^2 V_2^2}{a_1^2}}$$

$$V_2^2 = 2gh + \left(\frac{a_0}{a_1}\right)^2 C_c^2 V_2^2$$

$$V_2^2 \left[1 - \left(\frac{a_0}{a_1}\right)^2 C_c^2\right] = 2hg$$

$$V_2 = \sqrt{2gh} \Big/ \sqrt{1 - \left(\frac{a_0}{a_1}\right)^2 C_c^2}.$$

$$\text{Discharge } Q = V_2 \times a_2 = V_2 \times a_0 C_c = \frac{a_0 C_c \sqrt{2gh}}{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2 C_c^2}}.$$

The above expression is simplified by using —

$$C_d = C_c \frac{\sqrt{1-(a_0/a_1)^2}}{\sqrt{1-\left(\frac{a_0}{a_1}\right)^2 C_c^2}} \quad , \quad C_c = C_d \frac{\sqrt{1-\left(\frac{a_0}{a_1}\right)^2 C_c^2}}{\sqrt{1-\left(\frac{a_0}{a_1}\right)^2}}$$

Substituting this value of C_c in eqn (iv) we get —

$$Q = a_0 \times C_d \frac{\sqrt{1-(a_0/a_1)^2 C_c^2}}{\sqrt{1-\left(\frac{a_0}{a_1}\right)^2}} \times \frac{\sqrt{2gh}}{\sqrt{1-\left(\frac{a_0}{a_1}\right)^2 C_c^2}}.$$

$$= \frac{C_d a_0 \sqrt{2gh}}{\sqrt{1-\left(\frac{a_0}{a_1}\right)^2}} = \frac{C_d a_0 a_1 \sqrt{2gh}}{\sqrt{a_1^2 - a_0^2}}$$

Where, C_d = Co. of discharge for Orifice meter.

Notes → $\boxed{\text{The value of } C_d \text{ is less as compared to Venturimeter.}}$

Flow through pipe chapter-5

Pipe:— Pipe is a closed conduit. generally of circular cross-section used to carry water or any other fluid.

- when the pipe is running full. the flow is under pr but if the pipe is not running full the flow is under pr [culverts sewer pipes]

Loss of Fluid friction:— The frictional resistance of a pipe depends upon the roughness of the inside surface of the pipe more the roughness more the resistance.

-> Friction is known as fluid friction and the resistance is known as frictional resistance.

According Froude:—

i) Frictional resistance varies with sq of the velocity.
ii) Frictional resistance varies with natural surface.
iii) Among varies laws, the Darcy-weisbatch formulla & chezy's formulla.

Loss of energy in pipes:— When a fluid is flowing through a pipe, the fluid experiences some resistance due to which some it energy is loss—

Energy losses:—

1) Major Energy losses — Due to friction it is calculated by a darcy-weisbatch formula ~~pipe~~ & chezy's formulla.

2) Minor Energy losses — Due to sudden expansion of pipe.
ii) Sudden contraction of pipe.
iii) Bend in pipe.
iv) pipe fittings etc.
v) An obstruction of pipe.

Darcy - weisbatch formulla:— The loss of head in pipes due to friction calculated by darcy weisbatch eqn. $\boxed{h_f = \frac{4flv^2}{2gd}}$

where h_f = loss of head due to friction.
f = Co. of friction [Function of Reynolds number]

$= 16/R_e$ For $R_e < 2000$ [Viscous Flow]

$= \dfrac{0.079}{R_e^{1/4}}$ For R_e Varying From 4000 to 10^6.

L = Length of the pipe.
V = Mean vel of flow.
D = dia of the pipe.

Chezy's Formulla:—

$$h_F = \frac{F'}{8g} \times \frac{P}{A} \times L \times V^2$$

h_F = loss of head due to Friction.
P = wetted perimeter of pipe.
A = C.S area of pipe.
L = Length of pipe.
V = Mean vel of Flow.

$$M = \frac{A}{P} = \frac{\text{area of flow}}{\text{perimeter.}} = \text{Hydraulic mean depth or hydraulic radius.}$$

$$M = \frac{A}{P} = \frac{\frac{\pi}{4} d^2}{\pi d} = d/4.$$

Substituting $P/A = 1/M$, $h_f = \dfrac{F'}{8g} \times \dfrac{1}{M} \times L \times V^2$

$$V^2 = h_F \times \frac{8g}{F'} \times M \times \frac{1}{L}$$

$$V = \sqrt{\frac{Fg}{F'} \times M \times \frac{h_F}{L}}.$$

$\sqrt{Fg/F'} = C$, where C is Constant known as chezy's Constant.

$h_f/L = i$ loss of head per Unit length.

Substituting $V = C\sqrt{Mi}$ Value of M is always $d/4$.

Hydraulic gradient line:—

i) It is defined as the line which gives the sum of pr head $\frac{P}{w}$ & datum head (Z).

ii) If a Flowing Fluid in a pipe w.r.t the reference line or it the line which is obtained by joining of the top of the Vertical Coordinate showing pr head (P/w) of a Flowing Flui in a pipe from the Centre of the pipe.

iii) it is briefly written as H.G.L.

Total Energy line:—

i) it is defined as the line which gives the sum of pr head, dat head & kinetic head. of a flowing fluid w.r.t to some reference l or it is the line which obtained by joining the tops of all Vertical Coordinates showing Sum of

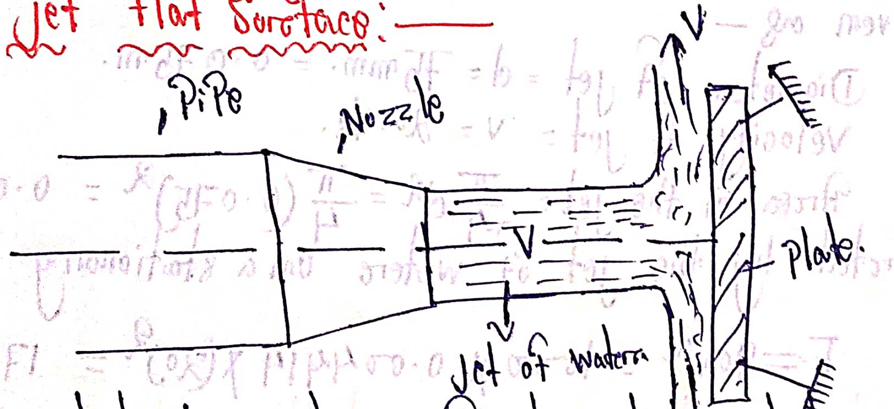
Impact of Jets — Chapter-6

Introduction — Impact of jet means the force exerted by the jet on a plate which may be stationary or moving.

Various Cases of Impact of jet are—

1. Force exerted by the jet on a stationary plate when—
   i) plate is vertical to the jet.
   ii) plate is inclined to the jet.
   iii) plate is curved.
2. Force exerted by the jet on a moving plate when—
   i) plate is vertical to the jet.
   ii) plate is inclined to the jet.
   iii) plate is curved.

Impact of jet Flat Surface:—



Force exerted by jet on fixed vertical plate:—

Consider a jet of water coming out from the nozzle strike a flat vertical plate—

Let V = Vel of the jet.
d = dia of the jet.
a = area of cross section of the jet $= \frac{\pi}{4} d^2$.

As the plate is fixed, the jet after striking will get deflected through 90°.

Hence the component of the vel of jet, in the dirn of jet after striking will be zero.

F_x = Rate of change of momentum in the dirn of force —

$$= \frac{\text{Initial momentum} - \text{Final Momentum}}{\text{time}}$$

$= \frac{\text{Mass} \times \text{Initial Vel} - \text{mass} \times \text{Final Vel}}{\text{Time}}$

$= \frac{\text{Mass}}{\text{Time}} [\text{Initial Vel} - \text{Final Vel}]$

$= \frac{\text{Mass}}{\text{sec}} [\text{Vel of jet before striking} - \text{Vel of jet after striking}]$

$= \rho a V [V - 0]$

$= \rho a V^2.$

Note:– In the above eqn Initial Vel minus Final Vel is taken as bcz force by the jet on the plate is calculated if force exerted on this jet is to calculated then final vel is taken.

Ex.1 Find the force exerted by a jet of water of dia 75 mm on a stat Flat plate when the jet strikes the plate normally with a vel of 20 m/s.

Soln → Data given as –

Diameter of jet $= d = 75\text{mm} = 0.075\text{m}$.

velocity of jet $= V = 20 \text{m/s}$.

Area of the jet $= \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.075)^2 = 0.004414 \text{m}^2$.

The force exerted by the jet of water on a stationary vertical pl is given by –

$F = \rho a V^2 = 1000 \times 0.004414 \times (20)^2 = 1766.8 \text{N}.$

Ex.2 Water is flowing through a pipe at the end of which a noz is fitted the dia of the nozzle is 100m. and the head of the water the centre of nozzle 100m. Find the Vel Force exerted by the jet wa on a fixed vertical plate. The Co. of Vel is 0.95.

Soln → Data given as:–

Diameter of nozzle $= d = 100\text{mm} = 0.1\text{m}$.

Head of water $H = 100\text{m}$.

Co. of Vel $C_v = 0.95$.

Area of nozzle $(a) = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.1)^2 = 0.007854 \text{m}^2$.

Theoritical Vel of jet of water is given as $V_{th} = \sqrt{2gH}$.

$= \sqrt{2 \times 9.81 \times 100} = 44.294 \text{m/s}.$

But $C_v =$ actual Vel / theoritical Velocity

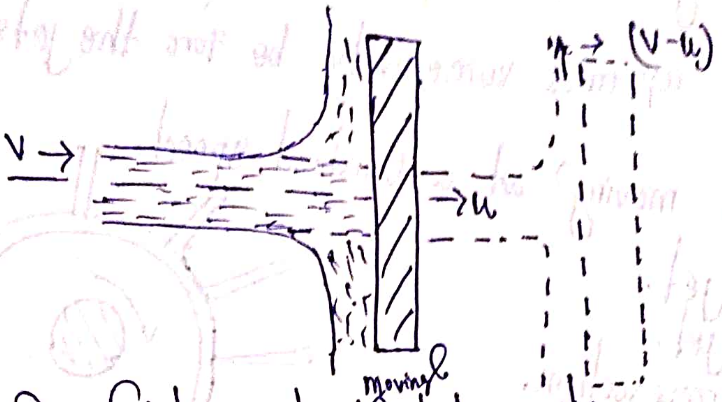
Actual vel of jet of water (V) = $C_v \times V_{th}$

$V = 0.95 \times 44.294 = 42.08$ m/s.

Force exerted on a fixed vertical plate is given by —

$F = \rho a V^2 = 1000 \times 0.07854 \times (42.08)^2 = 13907.2\text{ N} = 13.9072\text{ kN}$. Ans

Impact of jet on ...



Jet striking a flat vertical moving plate : —

Consider a jet of water striking a flat plate moving with a uniform vel away from the jet —

Let V = Vel of the jet.

a = Area of cross-section of the jet.

u = Uniform vel of flat plate.

In this case the jet strikes the plate with a rel velocity which is equal to the abs vel of jet of water minus the vel of the plate.

Hence rel vel of the jet with respect to plate = $V - u$.

Mass of water striking the plate per sec —

$= \rho \times$ area of the jet $\times$ Velocity $= \rho a \times [V - u]$

Force exerted by the jet on the moving flat plate in the dirn of motion of jet —

F_x = Mass of water striking/sec × [Initial Vel − Final Vel]

$= \rho a[V-u][(V-u)-0] = \rho a[V-u]^2$ [Final vel in the dirn of jet is zero.]

In this case the work will be done by the jet on plates as the plate is moving. Work done per sec by the jet on the plate —

$= \text{Force} \times \dfrac{\text{distance in the dirn of force}}{\text{time}}$

$= F_x \times u$

$= \rho a (V-u)^2 \times u$

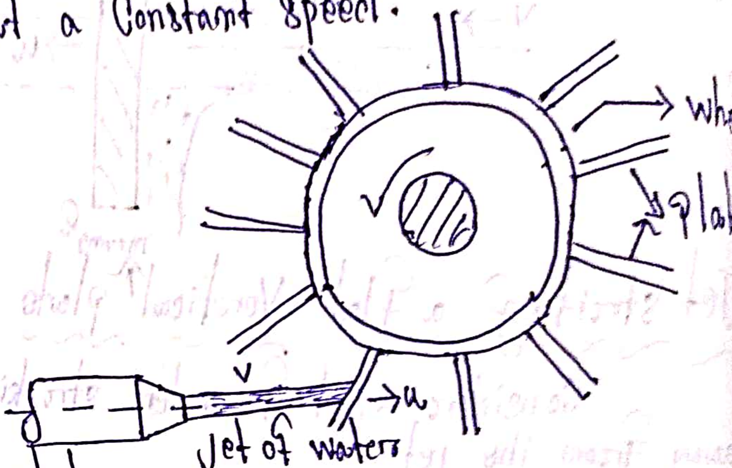
∴ | Flat plate Work done is zero

Jet strikes a Series of plate:— In this Case a large no of Flat p
are mounted On the rim of a wheel Fixed distance apart.

→ The jet strikes a plate and due to the Force exerted by the jet On the
the wheel starts moving and the 2nd plate mounted On the wheel app
before the jet. which again exerts the force On 2nd plate.

Thus each plate appears successively be fore the jets the jet e
Force On each plate.

The wheel starts moving at a Constant speed.



Let V = Vel of the jet.
D = dia of the jet.
A = Area of Cross-section.
u = Vel of plate

In this Case the mass of water Coming out from nozzle per Sec is always In Connect with the plates when all the plates are Considered.

→ Hence mass of water/sec = ρaV.

The jet strikes the plate with Vel = $V-u$

The force exerted by the jet In the dirⁿ of the motion of plate

$$F_x = \frac{\text{mass}}{\text{Time}}\,[\text{Initial Vel} - \text{Final Vel}]$$

$$= \rho aV\,[(V-u)-0]$$

$$= \rho aV\,[V-u]$$

Workdone per Second by the jet On the Series of the plate per Sec

$$= F \times u = \rho aV[V-u]\,u$$

Kinetic energy of the jet per Second $\rightarrow \frac{1}{2}mV^2 = \frac{1}{2}(\rho aV)V^2 = \frac{1}{2}\rho aV^3$.

$$\text{Efficiency} = \frac{\text{Workdone/sec}}{\text{kinetic energy/sec.}}$$

$$= \rho aV(V-u)u \Big/ \tfrac{1}{2}\rho aV^3 = 2u(V-u)/V^2$$

Condition for max efficiency — For a given jet velocity v, the efficiency will be max when.

$$\frac{d}{du} = 0, \Rightarrow \frac{d\left[\frac{2u(v-u)}{v^2}\right]}{du} = 0$$

$$\Rightarrow \frac{d\left[\frac{2uv-u^2}{v^2}\right]}{du} = 0$$

$$\Rightarrow \frac{2v-4u}{v^2} = 0$$

$$\Rightarrow 2v-4u = 0 \Rightarrow \boxed{u = v/2}$$

Maximum Efficiency :—

Substituting the value of $v = 2u$.

we get max efficiency as $\eta_{max} = 2u[2u-u]/(2u)^2 = \frac{1}{2} = 50\%$.

Impact of jet on a moving curved plate when jet strikes tangentialy at one of the tips:—

Consider a jet of water striking a moving curved vane tangentially at one of its tips.

In this case as plate is moving, the vel with which jet of plate is equal to the rel. vel of the jet w.r.t the plate.

Let V_1 = Vel of the jet at Inlet.

u_1 = Vel " " plate " "

V_{r1} = Rel vel of the jet & plate at Inlet.

α = guide blade angle.

θ = Vane angle made by rel vel V_{r1} with the dirn of motion of Inlet.

V_{w1} & V_{f1} = Components of V_1 in the dirn of motion & perpendicular to the dirn of motion of vane respectively.

V_{w1} = whirl vel at Inlet.

V_{f1} = vel at Inlet.

V_2 = Vel of jet at outlet.

u_2 = vel of ~~plate~~ vane. at outlet.

V_{r2} = Rel vel of jet at outlet.

β = Angle made by vel V_2 with dirn of motion of vane at outlet.

ϕ = Vane angle at outlet.
V_{w2} = Vel of whirl at outlet.
V_{r2} = Vel at outlet.
The triangle ABD & EGH are called the vel triangle at Inlet & outlet.
If the vane is smooth & having vel in the dirn of motion at Inlet & ou
equal we have —

$$U_1 = U_2 = U, \quad V_{r1} = V_{r2}.$$

Now, Mass of water striking vane per sec
Where a = Area of the jet.
V_{r1} = Rel vel at Inlet.
Force exerted by the jet in the dirn of motion —

$$F_x = \frac{\text{mass}}{\text{Time}} [\text{Initial Vel} - \text{Final Vel}]$$

But Initial vel with which jet strikes the vane = V_{r1}.
The component of this vel in the dirn of motion = $V_{r1} \cos\theta = (V_{w1} - U_1)$
Similarly the component of rel vel V_{r2} at outlet in the dirn of motion
$= -V_{r2}\cos\phi = -[U_2 + V_{w2}]$ [−ve sign is taken as compone V_{r2} in opposite dirn]
Substituting these values in the above eqn —

$$F_x = \rho a V_{r1}[(V_{w1} - U_1) - [-(U_2 + V_{w2})]]$$
$$= \rho a V_{r1}[V_{w1} - U_1 + U_2 - V_{w2}]$$
$$= a V_{r1}[V_{w1} + V_{w2}]$$

This eqn is true only when B is acute when —

$B = 90°$, $V_{w2} = 0$, $F_x = a V_{r1}[V_{w1}]$

When $B > 90°$ (obtuse) $F_x = a V_{r1}[V_{w1} - V_{w2}]$

In eqn F_x is written as $F_x = a V_{r1}[V_{w1} \pm V_{w2}]$

Work done /sec on the vane by the jet —

$$F_x \times U = \rho a V_{r1}[V_{w1} \pm V_{w2}] \times U$$

Workdone /sec unit weight of fluid striking/sec —

$$= \frac{\rho a V_{r1}[V_{w1} \pm V_{w2}] \times U}{\rho a V_{r1} \times g} = \frac{[V_{w1} \pm V_{w2}] \times U}{g}$$